FOPRA - 75 Particle physics with the computer

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1 Introduction

Particle Physics took huge steps in the couple of last centuries. In the Beginning one was able to analyse the gathered data by hand but nowadays such an attempt is impossible by any means. The data rates skyrocketed up to 50 million events per second, with a data acquisition times for several weeks. In order to process such a dense information one has to rely on computers. The goal for this experiment is to obtain interesting results from a huge amount of collected information from the HADES experiment in Darmstadt with help of elaborated algorithms implemented in a computer program.

2 Theory

2.1 Four-vectors

The Spectrometer used in the HADES experiment helps reconstructing the four-momentum of a particle:

$$p = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} \quad \text{whereby} \quad E = \sqrt{p^2 + m^2}. \tag{1}$$

So to get this four-vector one needs the three momentum components, which are measured by the spectrometer, and the mass of the specific particle to obtain the Energy. Therefore one has to perform particle identification in order to know which mass to use.

2.2 Particle Identification

The energy loss of a particle travelling through a material can be described by the Bethe-Bloche formula:

$$-\frac{dE}{dx} = \frac{4\pi n z^2}{m_e c^2 \beta^2} \cdot \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \cdot \left[\ln\left(\frac{2m_e c^2 \beta^2}{I \cdot (1-\beta^2)}\right) - \beta^2\right], \tag{2}$$

where $b = v/c$, v the velocity of the particle, E the energy of the particle, x the distance traveled by the particle, c speed of light, $z \cdot e$ the particle charge, e the elementary charge, m_e the rest mass of the electron, n the electron density of the traversed material and I the mean excitation potential of material.

2.3 Missing Mass

Some particles might not be detected by the MDC, for example because of their short life span. Those particles can still be reconstructed by applying the missing mass method:

$$M_{Missed} = (P_{Beam} + P_{Target} - P_1 - P_2 - ... - P_n)^2, \tag{3}$$

where $P_{1..n}$ are the masses of the detected particles.

2.4 Kinematics of Elastic Collisions

When dealing with elastic collisions, we know that energy and momentum are conserved. When considering a collision of a beam particle P_B with $|\vec{p_B}| = |p_{B,z}|$, a target particle P_T with $\vec{p_T} = 0$ and two particles P_1, P_2 after the collision we find:

$$\vec{p_{1,x}} + \vec{p_{2,x}} = 0 \tag{4}$$

$$\vec{p_{1,y}} + \vec{p_{2,y}} = 0 \tag{5}$$

$$\vec{p_{1,z}} + \vec{p_{2,z}} = \vec{p_{B,z}} \tag{6}$$

When applying conservation of momentum in the *center-of-mass (CM)* system in spherical coordinates we find:

$$\theta_1^* = \pi - \theta_2^* \tag{7}$$

$$\varphi_1^* = \pi - \varphi_2^*. \tag{8}$$

To transform the particle beam into the CM system, a Lorentz boost in z-direction is needed:

$$\tan\theta_1 = \frac{|p_1| \cdot \sin\theta_1}{|p_1| \cdot \cos\theta_1} = \frac{|p_1^*| \cdot \sin\theta_1^*}{\gamma_{pp}|p_1^*| \cdot \cos\theta_1^* - \gamma_{pp}\beta_{pp}E_1^*} = \frac{\sin\theta_1^*}{\gamma_{pp}(\cos\theta_1^* - \frac{\beta_{pp}}{\beta_1^*})}. \tag{9}$$

Using equation 7 we get a similar expression for θ_2:

$$\tan\theta_2 = \frac{\sin\theta_1^*}{\gamma_{pp}(-\cos\theta_1^* - \frac{\beta_{pp}}{\beta_1^*})}. \tag{10}$$

As β is defined as p/E one can obtain that $\frac{\beta_{pp}}{\beta_1^*} = 1$. With that, equation 9 and 10 can be connected and we obtain the *θ-condition*:

$$\tan\theta_1 \cdot \tan\theta_2 = \frac{1}{\gamma_{pp}^2} \tag{11}$$

Since we know that φ is the same in the CM and the lab-system, we also get a condition for φ from equation 8:

$$\varphi_1 + \varphi_2 = \pi \tag{12}$$

2.5 Reaction cross section

The reaction cross section is a value used in particle physics to characterize the probability of the particles interaction and is defined as

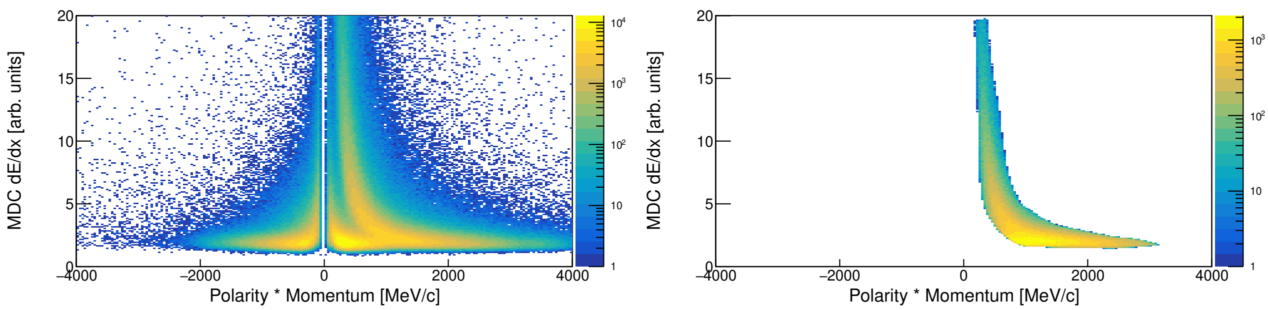
$$\sigma = \frac{R_{reac}}{\Phi} = \frac{\text{reaction rate for one target particle}}{\text{particle flux}} \tag{13}$$

The unit of a cross section is conventionally given in barn: $1\,\mathrm{b} = 10^{-28}\,\mathrm{m}^2$.

3 Procedure and Results

3.1 Selecting elastic collision events

In order to perform the given analysis tasks on the HADES data set one has to purge the data. To do so, one has to get specifically the data of the elastic collisions.

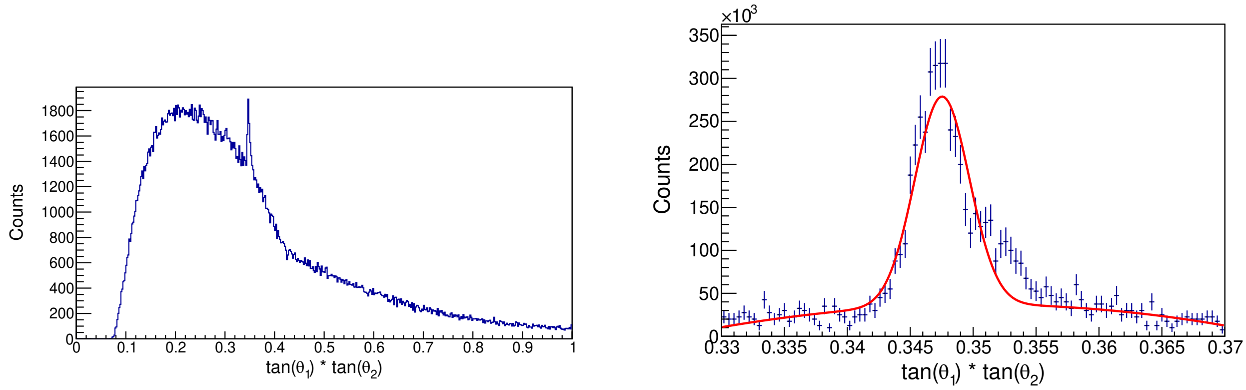


(a) before cut, all events detected by the experiment (b) after cut, only events where protons were detected

Figure 1: The energy loss (dE/dx) plotted against the particles momentum times their charge.

First we want to filter out events that did not produce protons. To do that, we apply a cut on the events in figure 1a, where the energy loss is plotted against the particles momentum times their charge.

The proton-proton events are represented by the yellow curve on the right, the cut is displayed in figure 1b.
Based on this set one proceeds with the actual data analysis. First of all, one needs to calculate the four-vector for all the events before and after the hit. With those vectors one can easily obtain information about energy, momentum as well as the angles θ and φ. From there on one creates several different histograms which lead us to the next step, to sieve out only a certain area of the histograms which are relevant to us, meaning filtering with a second layer to just to be left with the elastic collisions events. Figure 2b shows the number of events plotted against $\tan\theta_1 \cdot \tan\theta_2$. We know from

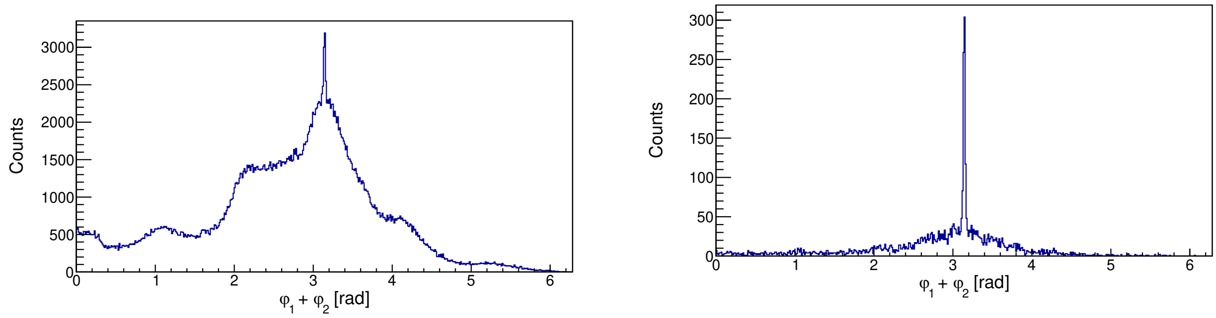


(a) before filtering

(b) after filtering events that do not satisfy the *θ-condition*(11) and the *φ-condition*(12), with fitted curve

Figure 2: Number of events vs. $\tan\theta_1 \cdot \tan\theta_2$

the θ-condition (11) that $\tan\theta_1 \cdot \tan\theta_2 = \frac{1}{\gamma^2}$. The latter can be easily calculated to $\frac{1}{\gamma_{pp}^2} = 0,349024$ for a pp-event with the given kinetic energy $E_{pp} = 3{,}5\,\mathrm{GeV}$. To filter for elastic events, we only take the events from the peak at $0,349024$ in figure 2b.



(a) before filtering

(b) after filtering events that do not satisfy the *θ-condition*(11)

Figure 3: Number of events vs. $\varphi_1 + \varphi_2$

The next step is to filter the events that satisfy the φ-condition. These are the ones from the peak at π in figure 3a, where the number of events that satisfy the θ-condition is plotted against $\varphi_1 + \varphi_2$. With that, we have now filtered our data set for elastic pp-collisions only.

3.2 Evaluation of the selection

Figure 6b shows the calculated missing mass and figure 7b the calculated missing energy for the selected events. Ideally, we should only have selected elastic events so the there should be no events with missing mass or energy left. We can see that only few of those events slipped into our selection, which means that our conditions are good enough. We could further reduce the number of events with missing mass by applying making the conditions more strict, but we would loose a lot of data in the process.

3.3 Getting the number of elastic events

The number of elastic events can now be acquired by fitting a curve to the selected data. The fitted curve is shown in figure 2b. From that, we get $N_{pp} = 1329 \pm 48$.

3.4 Getting the number of $p + p \rightarrow p + p + \pi^0$ events

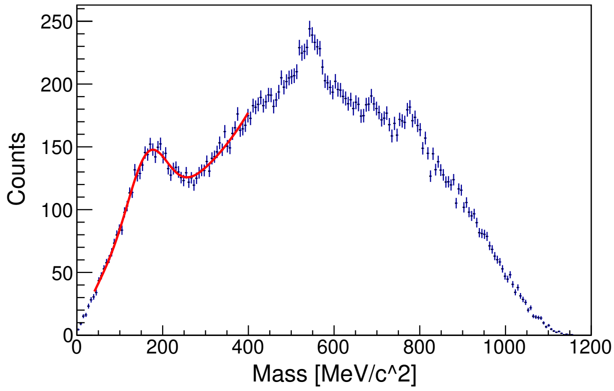


Figure 4: Number of inelastic collisions vs missing mass. The fitted peak resembles the collisions where a π_0 was produced.

To identify events where a π^0 was created, we can use the missing mass. Figure 4 shows the number of non-elastic events against the missing mass. We can see a peak at roughly the mass of a π^0, which is 134,98 GeV/c^2 [1]. The number of $p + p \rightarrow p + p + \pi^0$ event can once again be obtained by fitting the peak and we get $N_{pp\pi^0} = 5683 \pm 10$.

3.5 Estimation of a cross-section of the $p + p \rightarrow p + p + \pi^0$ reaction

Assuming that the acceptance of the events of $p + p$ and $p + p + \pi^0$ reactions are the same, following correlation between the cross sections $(\sigma_{pp}, \sigma_{pp\pi^0})$ and the numbers of observed collisions $(N_{pp}, N_{pp\pi^0})$ for both reactions can be used to determine an approximate value for the cross section of the of the $p + p \rightarrow p + p + \pi^0$ reaction:

$$\frac{N_{pp}}{N_{pp\pi^0}} = \frac{\sigma_{pp}}{\sigma_{pp\pi^0}} \tag{14}$$

The Assumption is valid in this case, because the High Acceptance Di-Electron Spectrometer (HADES) has a high acceptance as its name suggests. That means that the ratio of measured events and actual events are comparable for both events leading to equation (14).
The numbers of observed collisions are already known for both reactions from the previous tasks, so only the value of the cross section of the $p+p$ reaction σ_{pp} has to be determined in order to get $\sigma_{pp\pi^0}$. For computing the value a set of measured data on proton-proton elastic cross section σ_{pp} at a certain beam momentum p_{lab} in the laboratory frame was included in the user guide [2] for this experiment. In order to interpolate the elastic cross section at the beam momentum p_{beam} of the analyzed experiment the given data was fitted with the power function

$$\sigma_{pp} = a \cdot p_{\text{lab}}^{b} \tag{15}$$

and plotted in figure (5).

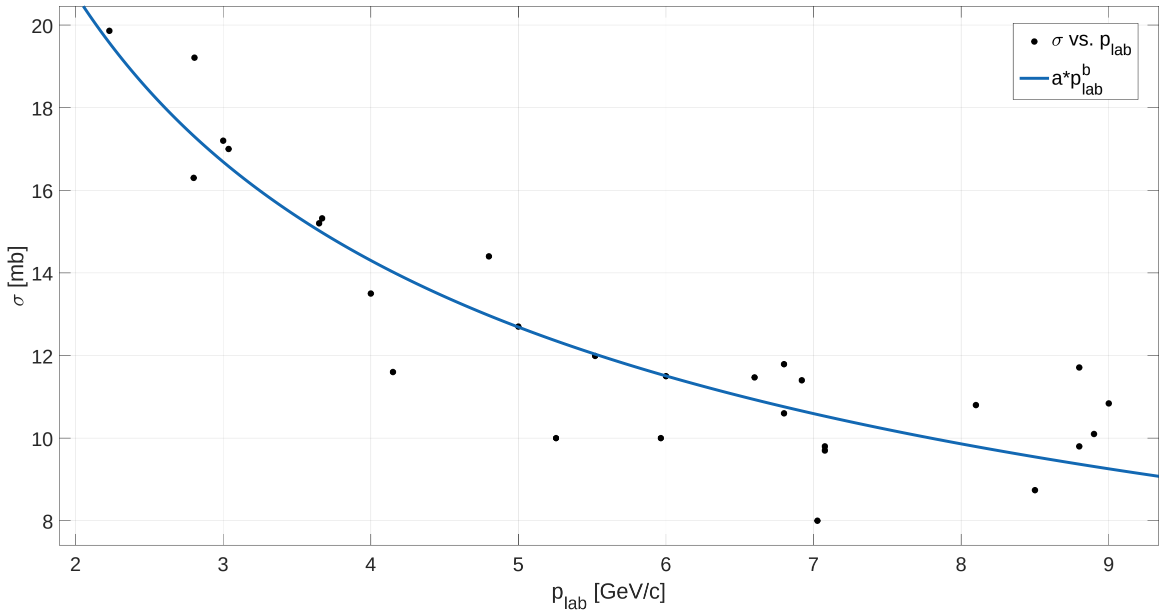


Figure 5: fit of the given data: σ vs. p_{lab}

With the the fit parameters $a = (30{,}08 \pm 4{,}20)$ and $b = (-0{,}5364 \pm 0{,}0090)$ that were obtained and the beam momentum $p_{beam} = 4{,}337\,96\,\mathrm{GeV/c}$ formula (15) was used to calculate $\sigma_{pp}(p_{beam}) = (13{,}69 \pm 2{,}63)\,\mathrm{mb}$.
Now that all values are known, $\sigma_{pp\pi^0}$ can be determined $\sigma_{pp\pi^0}$ from formula (14). Using the elaborated values for N_{pp}, $N_{pp\pi^0}$ and $\sigma_{pp}(p_{beam})$ one gets

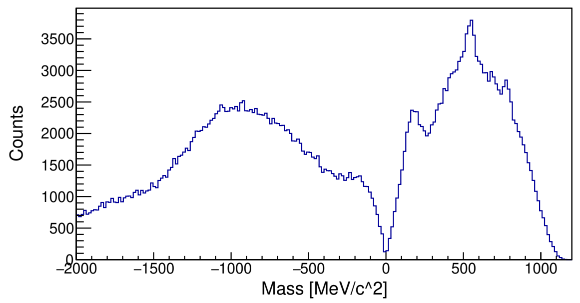
$$\sigma_{pp\pi^0} = \frac{\sigma_{pp} \cdot N_{pp\pi^0}}{N_{pp}} = (62{,}42 \pm 6{,}78)\,\mathrm{mb},$$

which is about 4.5 times larger than the elastic cross section.

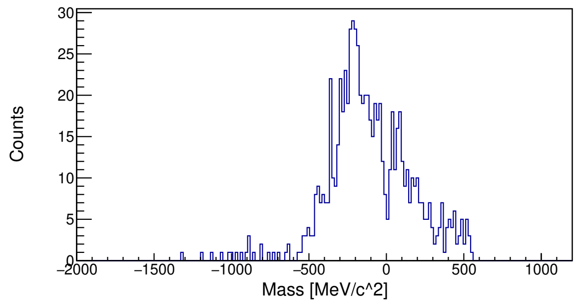
4 Conclusion

We saw that with few assumptions and the help of computer programming, we can precisely extract the data necessary for our task from a large data set. In our case, we could calculate the cross sections of elastic and π_0-inelastic proton-proton collisions with some basic understanding of elastic collisions and the help of the missing mass method.

5 Appendix

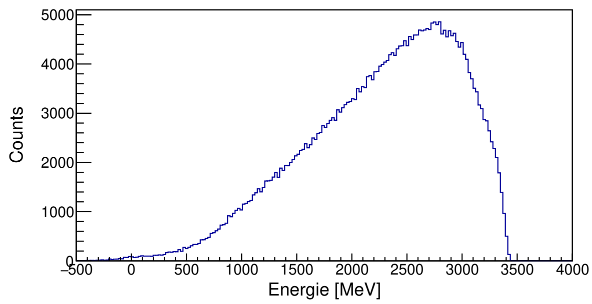


(a) before filtering

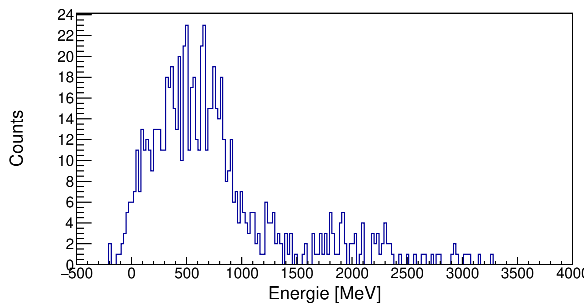


(b) after filtering events that do not satisfy the θ-*condition* (11) and the φ-*condition* (12)

Figure 6: Number of events vs. missing mass

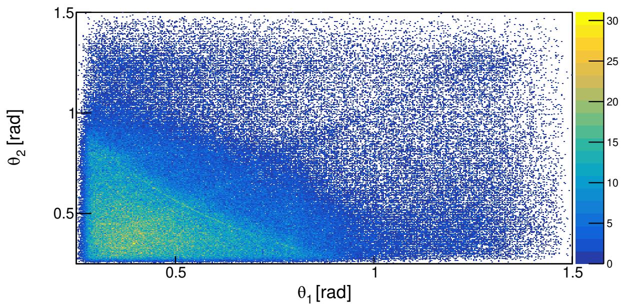


(a) before filtering

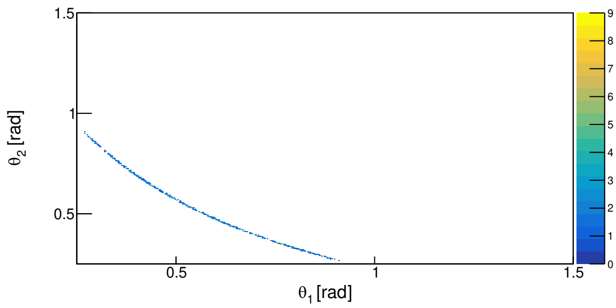


(b) after filtering events that do not satisfy the θ-*condition* (11) and the φ-*condition* (12)

Figure 7: Number of events vs. missing energy



(a) before filtering



(b) after filtering events that do not satisfy the θ-*condition*(11) and the φ-*condition*(12)

Figure 8: θ_1 vs θ_2, color represents the number of events

References

[1] Particle Data Group. *2020 Review of Particle Physics*. 2020. URL: https://pdg.lbl.gov/2020/tables/contents_tables.html.

[2] Kirill Lapidus Eliane Epple. *Userguide: Particle physics with the computer*. 2011. URL: https://www.ph.tum.de/academics/org/labs/fopra/docs/userguide-75.en.pdf.