Lab Report on "Magnetism"

Basic Lab Course Part 2
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04. September 2020

Abstract

The aim of this experiment is to measure the magnetic field of a coil with and without a core. By doing so, the longitudinal and transversal position of the coil is regarded separately.

Contents

1 Basics

1.1 Biot-Savart law

An essential way to determine the magnetic field of a conductor is the law of Biot-Savart. It states that for an electric current running through $d\vec{l}$ the created magnetic field $d\vec{B}$ at the distance $\vec{r}$ is

$$d\vec{B} = \frac{\mu_0 I d\vec{l} \times \frac{\vec{r}}{r}}{4\pi r^2} \tag{1}$$

If we use this formula for a circular conductor and are interested in the magnetic field on the axis we find

$$B(x) = \frac{\mu_0}{4\pi} \frac{2\pi R^2 I}{(x^2 + R^2)^{3/2}}. \tag{2}$$

On a coil with N windings the magnetic fields of the single windings, that behave roughly like the ones of circular conductors, influence each other. If the windings are very close to one another, or in other words if the length of the coil is much smaller than the distances, in which the measurements are conducted, we can simply multiply N to equation 2. However if the length is much bigger than the coil's radius, we get

$$B = \mu_0 \frac{N}{L} I \tag{3}$$

For a coil with a core material of relative permeability μ_r formula 2 and 3 change to

$$B(x) = \mu_0 N \frac{\mu_r}{4\pi} \frac{2\pi R^2 I}{(x^2 + R^2)^{3/2}}. \tag{4}$$

$$B = \mu_0 \mu_r \frac{N}{L} I \tag{5}$$

1.2 Matter in the magnetic field

Different kinds of materials react differently to magnetic fields. Therefore they can be divided in mainly three categories: paramagnets, diamagnets and ferromagnets. The differences between those categories will be explained in the following. An important quantity for rating the interaction of a material with magnetism is the magnetisation $\vec{M}$. It describes the magnetic moment $\vec{m_m}$ per unit of volume.

$$\vec{M} = \frac{\vec{m_m}}{V} \tag{6}$$

If we now put a material in the middle of a coil, its magnetic field contains the sum of the magnetic field without material $\vec{B_0}$ and the magnetisation $\vec{M}$ of the material.

$$\vec{B} = \vec{B_0} + \mu_0 \vec{M} \tag{7}$$

To determine the relative permeability of a material one can use

$$\mu_r = \frac{B}{B_0}. \tag{8}$$

1.2.1 Paramagnetism

As mentioned earlier, paramagnetism is one of the three classes in which materials can be categorized regarding their interaction with magnetism. Paramagnets have permanent dipole moments, which are directed randomly in every direction. If a magnetic field is brought near the dipoles, some of them straighten themselves in the direction of the field. Therefore they strengthen it. The magnetic susceptibility is always positive and in a scale of 10^{-5}.

1.2.2 Diamagnetism

Diamagnets do not have permanent dipole moments. An external magnetic field will induce a circular current, that weakens the magnetic field. The magnetic susceptibility is always negative and in a scale of 10^{-5}.

1.2.3 Ferromagnetism

Ferromagnets interact strongly with magnetic fields. Those materials do not only have permanent dipole moments, but they are also already straightened in one direction in certain areas, the so-called Weiss domains. With no external magnetic field the domains are straightened in different directions, so that in total there is no magnetic field. However under the influence of an external field the domains with all the dipoles straighten in direction of the field. This phenomenon extremely strengthens the field, which means, that $\mu_r \gg 1$. Ferromagnetism is found in materials like iron, cobalt or nickel.

2 Experimental procedure

2.1 Magnetic field profile of a coil in longitudinal direction

In the first part of the experiment, a longitudinal Hall probe is installed on a movable sledge. The sledge itself is located on a rail so that its distance to the coil can be measured easily. After connecting the probe with a magnetometer, the axis of the coil is positioned so that it points in the same direction as the rail. This setup can be seen in figure 1.

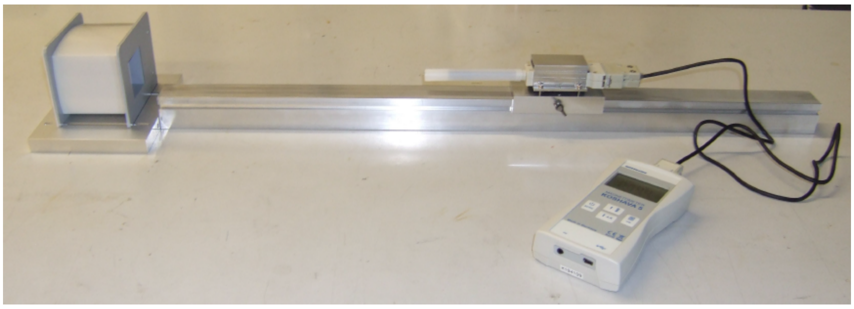

Figure 1: Experiment setup for measuring the magnetic field of a coil [2]

After that, you apply voltage to 1200 windings of the coil and adjust the current to 1A. During the measurements, the electricity can be switched on and of at will.
The next step is to find the spot on the rail where the magnetometer shows the highest value of the magnetic field while the current is flowing. Starting at this point, you slowly move the Hall probe away from the coil and write down the data of the magnetic field at each point with and without a current. Near the coil the values are measured at small steps in order to ensure a higher accuracy. Yet the nearer you come to the end of the rail, the fewer data are noted because the magnetic field does not change there significantly any more.
By determining the data for each distance also with the electricity switched off, it is possible to measure the magnetic background field.
These steps are repeated for a current of $0,5$A. For the last measurements in longitudinal direction you put a metal core inside the coil and change the electricity back to 1A.

2.2 Magnetic field profile of a coil in transversal direction

After turning the coil by 90 degrees, removing the core and replacing the probe by a transversal Hall probe, you follow exactly the same procedure as described above in order to measure the transversal magnetic field. The electricity is also adjusted to 1A.

3 Results and discussion

3.1 Magnetic field profile of a coil in longitudinal direction

Figure 2 shows the different data sets of the measured background field as well as the (resulting) magnetic field profile of the coil in longitudinal position depending on the distance. The horizontal and vertical lines represent the uncertainty $u(B)$ of each value. Yet the uncertainties for the distance $u(x)$ are not included in the plot in order to increase the clarity of the results.
As clearly visible, the extremely weak background field remains almost constant throughout the entire length of the rail. Its minimal fluctuations are probably caused by the terrestrial magnetic field and other technical devices standing nearby.
In contrast to that, the function of the resulting magnetic field descends drastically while the distance is getting larger. This behavior can be explained with equation 4. After having its peak at the center of the coil, the magnetic field converges to zero at the end of the rail.

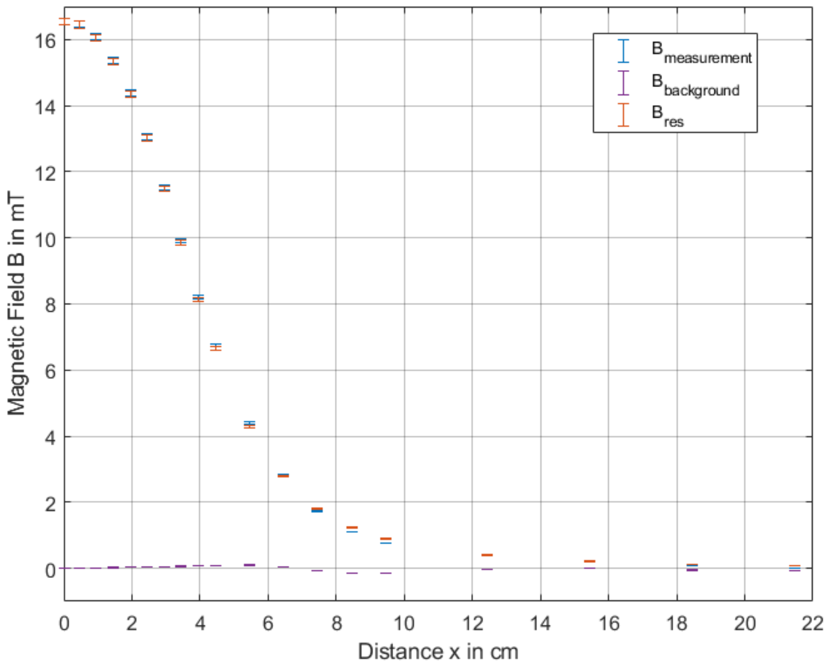

Figure 2: Coil in longitudinal orientation without core, $I = 1,00$ A

The depicted functions in figure 3 behave very similarly to the ones in figure 2. However, you can see that the values of the magnetic field depend on the current flowing through the coil. As shown in equation 4, the magnetic field is proportional to the electricity. Therefore, the values approximately halve themselves compared to figure 2.

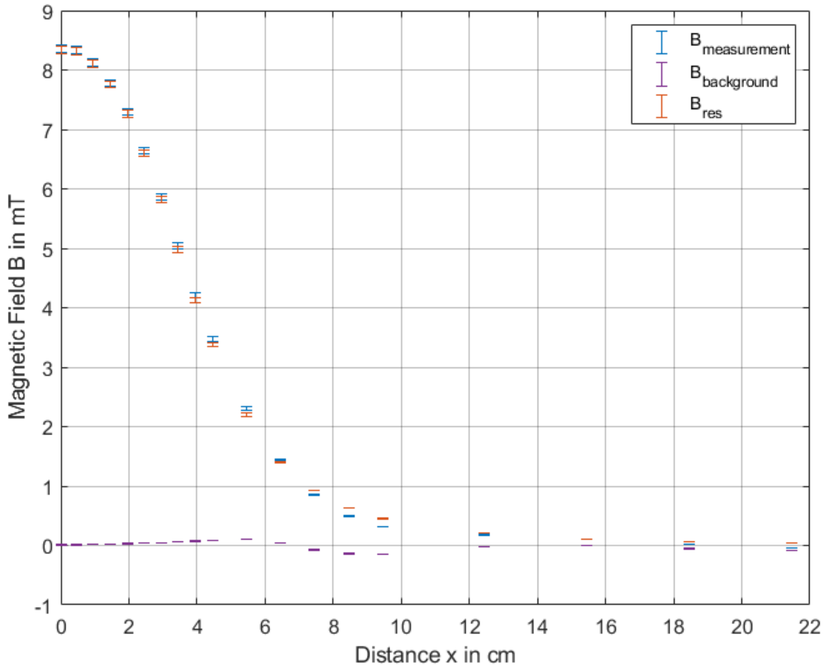

Figure 3: Coil in longitudinal orientation without core, $I = 0,500$ A

3.2 Determination of R_{eff} and I_{eff}

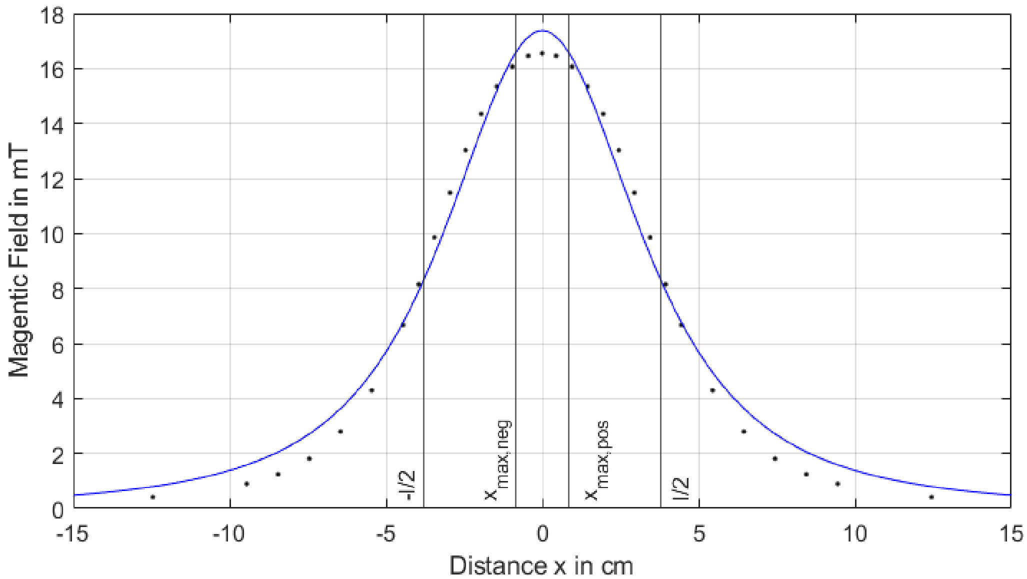

Figure 4: Coil in longitudinal orientation without core and fitted Biot-Savart, $I = 1,00$ A

In figure 4 the measured data have been flipped vertically at the center of the coil in order to create an impression of the complete magnetic field. By fitting the data according to equation 4 and using R and I as free parameters, one can determine $R_{\text{eff}} = 4.756$ cm and $I_{\text{eff}} = 1.095$ A.

Since the coil in this experiment is quadratic and not round, R_{eff} is an approximation on how big the effective radius of the same coil with a circular opening would be. Furthermore, you cannot simply use $I = 1,00$A for your calculations as the value of the current changes due to alternations of the resistance of the coil. Therefore the item I_{eff} is needed, too.
Even though the function in figure 4 apparently reaches a peak in the center of the coil, in reality the magnetic field would remain constant there. The Biot-Savart law used in the fitted curve is only an approximation, since it does not include the length of the coil. This causes the difference between the measured data and the curve. In order to determine the limits $x_{\text{max,neg}}$ and $x_{\text{max,pos}}$ of this constant area, equation 4 is solved for x. If you insert the highest measured value for the magnetic field $B_{\text{max}} = 16,56$ mT in the resulting formula

$$x = \sqrt{\left(\frac{2\pi\mu_r\mu_0 N R_{\text{eff}}^2 I_{\text{eff}}}{4\pi B_{\text{max}}}\right)^{\frac{2}{3}} - R_{\text{eff}}^2} \tag{9}$$

you get $x_{\text{max,neg}} = -0.859 \pm 0.056$ cm and $x_{\text{max,pos}} = 0.859 \pm 0.056$ cm.

3.3 Determination of the core-material

The measurement for the magnetic field with a core inside the coil shows about 2.5 times higher values compared to the one without the core and same current. This indicates that the core consists of a ferromagnetic material. To determine the exact value of the relative permeability μ_r, the data points are mirrored at the y-axis and fitted with the Biot-Savart law (equation 4) as shown in figure 5. Therefore, the already determined values for R_{eff} and I_{eff} are used. The fit gives $\mu_r = 2.73$, which is consistent with the previous observation. We could not find a ferromagnetic material with a value this low. Since the relative permeability of ferromagnetic materials is not constant, but rather dependant of the magnetic field it is in, this does not mean the value is wrong. By the looks, the core was probably made of iron. Iron reaches its saturation point in a magnetic field of about 1-2 Tesla [1]. But since the magnetic field of the coil alone was about 17 mT, the iron was way beneath the saturation point and therefore, it had not reached maximum relative permeability.

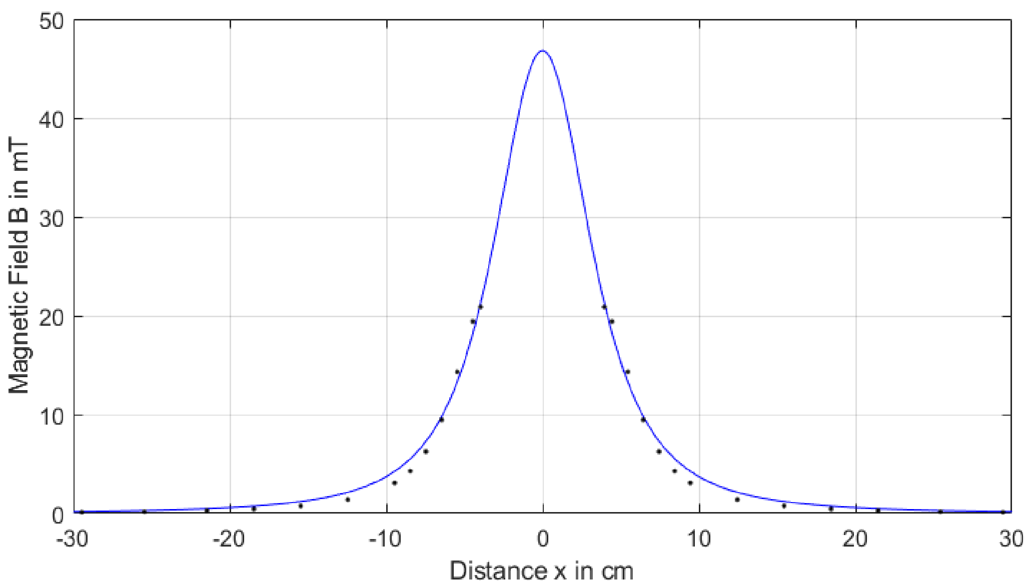

Figure 5: Coil in longitudinal orientation with core, $I = 1,00$ A

By using $\mu_r = 2.73$ and $x = 0$ m in equation 4 the theoretical maximum value for the magnetic field can be calculated to $B_{\text{max}} = 47.36$ mT. The real value should be a bit smaller than this, since the

Biot-Savart law is only an approximation here.
To determine the magnetisation M of the core, equation 7 can be solved for M:

$$M = \frac{B - B_0}{\mu_0} \tag{10}$$

We use $B = B_{\text{max,core}} = 47.36$ mT and $B_0 = B_{\text{max,nocore}} = 16,56$ mT and get $M = 24510\,\frac{\text{A}}{\text{m}}$

3.4 Magnetic field profile of a coil in transversal direction

Figure 6 shows the resulting magnetic field of the coil in transversal orientation (background magnetic field is already subtracted). The curve is similar to the one in longitudinal orientation, but the values are much smaller. On the axis through the inner of the coil (longitudinal configuration), all sides of the coil contribute to the magnetic field, while in transversal configuration only the side facing the magnetometer contributes. This explains the smaller values.

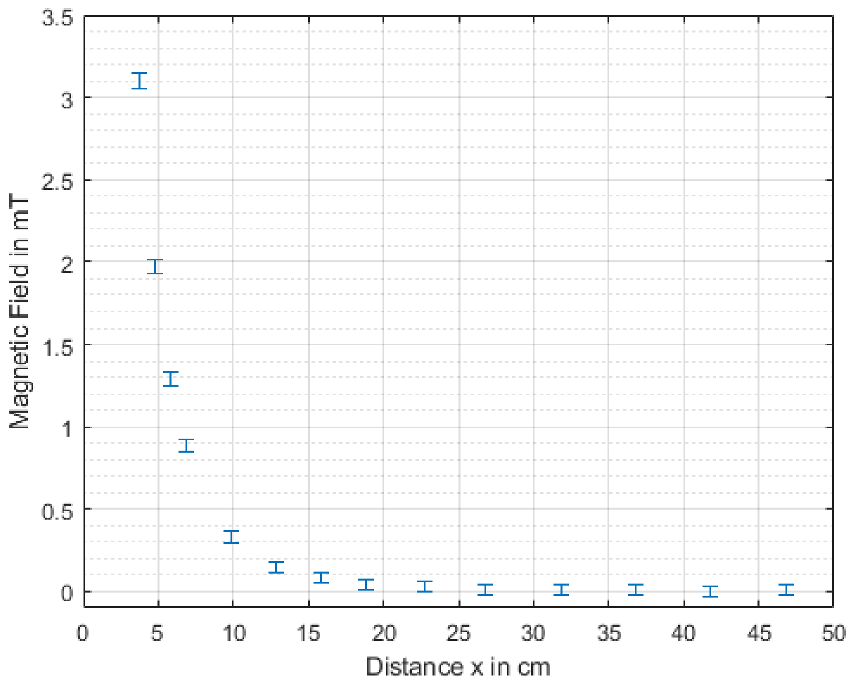

Figure 6: Coil in transversal orientation without core, $I = 1,00$ A

4 Bibliography

References

[1] *Investigation of measurement method of saturation magnetization of iron core material using electromagnet*. AIP Advances. URL: `https://aip.scitation.org/doi/10.1063/1.4993998`.

[2] Fakultät für Physik. *Magnetismus (MAG)*. Technische Universität München. June 23, 2020. URL: `https://www.ph.tum.de/academics/org/labs/ap/ap2/MAG.pdf`.